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New Tensor Interactions in μ Decay

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Abstract

The most general form of the hamiltonian for the muon decay is presented. We assume that it arises as a result of the exchange of intermediate bosons with a momentum q and naturally should depend on this momentum. That allows us to introduce two additional coupling constants for the tensor interactions which give rise to new parameters in the energy spectrum of positrons. The experimental consequences of such a generalization are discussed.

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1 Introduction

One of the basic decay processes in the weak interactions is the μ decay $\mu^+ \rightarrow e^+ \nu_e \tilde{\nu}_\mu$. Free of any QCD complications, it can be used for a thorough check of the standard theory of electroweak interactions and the determination of the Fermi coupling constant G_F . The experimental accuracy is so high that the one-loop electromagnetic radiative corrections must necessarily be taken into account. Therefore, if some nonstandard interactions are of the same order of magnitude, $O(\alpha)$, then they can be extracted on the background of the V–A interaction and radiative corrections to it. In the literature, the following most general form of the μ -decay hamiltonian is accepted [1]

$$\mathcal{H} = \frac{4G_F}{\sqrt{2}} \sum_{\substack{k=S,V,T \\ \epsilon, \chi=R,L}} \left\{ g_{\epsilon\chi}^k [\bar{e}_\epsilon \Gamma^k \nu_n^e] [\bar{\nu}_m^\mu \Gamma^k \mu_\chi] + \text{h.c.} \right\}. \quad (1)$$

Here, k labels the type of interaction (scalar, vector, tensor), ϵ and χ indicate the chirality of the charged leptons. (The chiralities of the neutrinos, n and m , are uniquely fixed by ϵ , χ , and k). The standard V–A interaction implies that $g_{LL}^V=1$, and other $g_{\epsilon\chi}^k$ are zero. Nonstandard couplings may arise in extensions of the standard model from the exchange of new intermediate bosons, other than W^\pm . At the first sight, it may seem that there are 12 (generally complex) constants $g_{\epsilon\chi}^k$. However, the local tensor interactions $[\bar{e}_R \sigma^{\alpha\beta} \nu_L^e][\bar{\nu}_L^\mu \sigma_{\alpha\beta} \mu_R]$ and $[\bar{e}_L \sigma^{\alpha\beta} \nu_R^e][\bar{\nu}_R^\mu \sigma_{\alpha\beta} \mu_L]$ are identically equal to zero. Therefore, the coupling constants g_{RR}^T and g_{LL}^T are absent in eq.(1), and the most general form of a local, Lorentz-invariant, derivative-free, and lepton-number-conserving four-fermion interaction is parameterized by 10 model-independent constants.

In the present paper we are going to demonstrate that, abandoning the locality of the *effective* Fermi interaction, we can introduce two additional constants g_{RR}^T and g_{LL}^T in front of the interaction terms which depend on the momentum transfer q_μ . Such terms arise from the exchange of tensor particles (the fundamental interactions of which are *local*) in an extended model of the electroweak interactions [2]. Let us redefine the tensor structure to be

$$\Gamma^T \otimes \Gamma^T \equiv \frac{1}{2} \sigma^{\alpha\lambda} \otimes \sigma_{\beta\lambda} \cdot \frac{4q_\alpha q^\beta}{q^2}. \quad (2)$$

Then the terms with g_{LR}^T and g_{RL}^T in eq.(1) remain the same, owing to the identity

$$\sigma^{\alpha\lambda} P_\pm \otimes \sigma_{\beta\lambda} P_\pm \cdot \frac{4q_\alpha q^\beta}{q^2} = \sigma^{\alpha\beta} P_\pm \otimes \sigma_{\alpha\beta} P_\pm,$$

where $P_\pm = \frac{1}{2}(1 \pm \gamma^5)$ is the chiral projection operator. In the most general case, one can assume that the effective four-fermion interaction arises from exchange of some bosons with a momentum q_μ (that is, the interaction depends only on the momentum transfer). Then, in fact, eq.(1) with the above definition (2) will be the most general form of the effective interaction of charged leptons (up to a factor depending on q^2) — the matrix structure $\gamma^\alpha \otimes \gamma^\beta \cdot q_\alpha q_\beta$ for the particles on the mass shell is reduced to the scalar structure $\Gamma^S \otimes \Gamma^S$.

The tensor particles have been introduced for the following reason. In the recent experiments $\pi^- \rightarrow e^- \tilde{\nu} \gamma$ [3] and $K^+ \rightarrow \pi^0 e^+ \nu$ [4], tensor form factors have been discovered. These form factors cannot be explained in the framework of the standard V–A interaction [5]. For the semileptonic weak decays, an additional interaction has been introduced [2]

$$\mathcal{L}_{qe} = -\sqrt{2} G_F f_t \bar{u} \sigma^{\alpha\lambda} d' \frac{q_\alpha q^\beta}{q^2} \bar{e}_R \sigma_{\beta\lambda} \nu_L, \quad (3)$$

where $d' = d \cos \theta_C + s \sin \theta_C$, and the value of f_t can be found from analyzing these meson decays. In the framework of QCD, by applying the PCAC technique [5], we obtain the value $f_t = (7.84 \pm 2.24) \times 10^{-2}$ [6] from the pion-decay data [3]. If we assume that the coupling constants of the tensor particles to quarks and leptons are the same, then we can determine the tensor constants in eq.(1): $g_{LR}^T = g_{RL}^T = g_{LL}^T = 0$ ³

$$g_{RR}^T = \frac{f_t}{4} = (1.96 \pm 0.56) \times 10^{-2}. \quad (4)$$

However, for the sake of generality, we consider the model-independent case with all $g_{ex}^T \neq 0$. Introducing the tensor coupling constants g_{LL}^T and g_{RR}^T leads to the appearance of new parameters in the e^+ energy spectrum. Below we discuss in detail the consequences for processing the experimental data on decays of nonpolarized muons and for determining the Fermi constant.

2 The positron energy spectrum

Generally speaking, one can assume the existence of right-handed neutrinos. Since the upper bound on the muon neutrino mass is only $m_{\nu_\mu} < 270$ keV [8], the effects of the neutrino mass could be comparable to that of m_e . The mass of ν_e is always neglected in this paper.

It is straightforward to calculate the e^+ energy spectrum for the μ^+ decay including all mass effects. Let us introduce the scaled e^+ energy $x_e = 2E/(\omega m_\mu)$ which varies in the interval $x_o \leq x_e \leq 1$, where $\omega = 1 + \epsilon_e^2 - \epsilon_\nu^2$, $x_o = 2\epsilon_e/\omega$, and $\epsilon_e = m_e/m_\mu$, $\epsilon_\nu = m_\nu/m_\mu$. Then the spectrum reads

$$\begin{aligned} \frac{d\Gamma}{dx_e} = & \frac{A G_F^2 m_\mu^5}{256\pi^3} \left[h_1 + \frac{2}{9} \rho h_2 + 4\epsilon_e^2 (1 - \epsilon_\nu^2) \tau h_3 + \epsilon_e (\eta h_4 + \varepsilon h_5) \right. \\ & \left. + \epsilon_\nu (\lambda h_6 + 4\epsilon_e^2 \nu h_7) + \epsilon_e \epsilon_\nu (\sigma h_8 - \kappa h_5) \right]. \end{aligned} \quad (5)$$

The functions h_1, \dots, h_8 are given by

³The pion decay puts very strong restrictions on the tensor couplings [7]. Elimination of the above three constants is enough to satisfy the constraints. That just means that, as well as in the standard weak interactions, only left-handed neutrinos take part in the tensor interactions.

$$\begin{aligned}
h_1 &= \sqrt{x_e^2 - x_o^2} (1 - x_e)^2 \frac{\omega^5 x_e}{u}, \\
h_2 &= \sqrt{x_e^2 - x_o^2} (1 - x_e)^2 \left\{ 4u^3 - u^2(5 + 5\epsilon_e^2 + \epsilon_\nu^2) + u \left[(1 - \epsilon_e^2)^2 - \epsilon_\nu^2(1 + \epsilon_e^2) \right] \right. \\
&\quad \left. + 2\epsilon_\nu^2(1 - \epsilon_e^2)^2 \right\} \frac{\omega^4}{u^3}, \\
h_3 &= \omega^2(2x_e - 1) \ln \frac{t_{\max}}{t_{\min}} - \left[1 + \frac{u}{u - p} \right] \frac{\omega^3(1 - x_e) \sqrt{x_e^2 - x_o^2}}{u}, \\
h_4 &= 2\sqrt{x_e^2 - x_o^2} (1 - x_e)^2 \frac{\omega^4}{u}, \\
h_5 &= \left[1 + \epsilon_e^2 + \epsilon_\nu^2 - 3u + \frac{\epsilon_\nu^2(1 - \epsilon_e^2)}{u} \right] \frac{\omega^3(1 - x_e) \sqrt{x_e^2 - x_o^2}}{u} - 2\epsilon_e^2(1 - \epsilon_\nu^2) \ln \frac{t_{\max}}{t_{\min}}, \\
h_6 &= \sqrt{x_e^2 - x_o^2} (1 - x_e)^2 \frac{\omega^4(1 - \epsilon_e^2 - u)}{u^2}, \\
h_7 &= \epsilon_e^2 \ln \frac{t_{\max}}{t_{\min}} - \frac{\omega^3(1 - x_e) \sqrt{x_e^2 - x_o^2}}{u}, \\
h_8 &= \sqrt{x_e^2 - x_o^2} (1 - x_e)^2 \frac{\omega^4(1 - \epsilon_e^2 + u)}{u^2},
\end{aligned}$$

where $u = 1 + \epsilon_e^2 - \omega x_e$, $p = \epsilon_\nu^2(1 - \epsilon_e^2 - \epsilon_\nu^2)/(1 - \epsilon_\nu^2)$, and

$$t_{\min}^{\max} = \frac{1}{2u} \left[u(1 + \epsilon_e^2 + \epsilon_\nu^2) - u^2 - \epsilon_\nu^2(1 - \epsilon_e^2) \pm (u - \epsilon_\nu^2) \sqrt{(1 - \epsilon_e^2)^2 - 2u(1 + \epsilon_e^2) + u^2} \right].$$

The Michel parameter ρ and the quantities η , λ , σ , τ , ε , ν , and κ are functions of the coupling constants $g_{\epsilon\chi}^V$:

$$\begin{aligned}
\rho &= \frac{3}{A} \left\{ \sum_{\epsilon, \chi=R,L} |g_{\epsilon\chi}^S - 2g_{\epsilon\chi}^T|^2 + 4|g_{LL}^V|^2 + 4|g_{RR}^V|^2 \right\}, \\
\eta &= \frac{8}{A} \sum_{\epsilon, \chi=R,L} \text{Re} \left\{ g_{\epsilon\chi}^V (g_{\bar{\epsilon}\chi}^S + 6g_{\bar{\epsilon}\chi}^T)^* \right\}, \\
\lambda &= \frac{8}{A} \text{Re} \left\{ g_{LL}^S g_{LR}^{S*} + g_{RR}^S g_{RL}^{S*} - 2g_{LL}^V g_{LR}^{V*} - 2g_{RR}^V g_{RL}^{V*} - 4g_{LL}^T g_{LR}^{T*} - 4g_{RR}^T g_{RL}^{T*} \right\}, \\
\sigma &= \frac{8}{A} \sum_{\epsilon, \chi=R,L} \text{Re} \left\{ g_{\epsilon\chi}^V (g_{\bar{\epsilon}\chi}^S - 6g_{\bar{\epsilon}\chi}^T)^* \right\}, \\
\tau &= \frac{16}{A} \left\{ |g_{LL}^T|^2 + |g_{RR}^T|^2 \right\}, \\
\varepsilon &= \frac{32}{A} \text{Re} \left\{ g_{LL}^V g_{RR}^{T*} + g_{RR}^V g_{LL}^{T*} \right\}, \\
\nu &= \frac{32}{A} \text{Re} \left\{ g_{LL}^T g_{LR}^{T*} + g_{RR}^T g_{RL}^{T*} \right\},
\end{aligned}$$

$$\kappa = \frac{32}{A} \text{Re} \left\{ g_{LR}^V g_{RR}^{T*} + g_{RL}^V g_{LL}^{T*} \right\},$$

where the bar denotes opposite chiralities, and

$$A = 4 \sum_{\epsilon, \chi=R,L} \left\{ |g_{\epsilon\chi}^S|^2 + 4 |g_{\epsilon\chi}^V|^2 + 12 |g_{\epsilon\chi}^T|^2 \right\}.$$

Notice that introducing new tensor constants leads to the appearance of new structures τ , ε , ν , and κ in the spectrum. This also gives rise to a change in the definition of the quantities ρ , η , λ , and σ in favor of a more symmetric form as compared to their standard definition [9].

Integrating over the whole spectrum, we can derive the partial decay width of the muon into a positron

$$\Gamma = \frac{A G_F^2 m_\mu^5}{256\pi^3} \left[H_1 + \epsilon_e \eta H_4 + \epsilon_\nu (\lambda H_6 + 4\nu \epsilon_e^2 H_7) + \epsilon_e \epsilon_\nu \sigma H_8 \right], \quad (6)$$

where $H_i = \int_{x_o}^1 h_i(x_e) dx_e$ ($i=1, \dots, 8$).

$$\begin{aligned} H_1 &= \frac{1}{12} R_o (1 - 7\epsilon_e^2 - 7\epsilon_\nu^2 - 7\epsilon_e^4 - 7\epsilon_\nu^4 + 12\epsilon_e^2 \epsilon_\nu^2 + \epsilon_e^6 + \epsilon_\nu^6 - 7\epsilon_e^4 \epsilon_\nu^2 - 7\epsilon_e^2 \epsilon_\nu^4) \\ &\quad + 2\epsilon_e^4 (1 - \epsilon_\nu^4) L_e + 2\epsilon_\nu^4 (1 - \epsilon_e^4) L_\nu, \\ H_2 &= H_3 = 0, \\ H_4 &= \frac{1}{3} R_o (1 + 10\epsilon_e^2 - 5\epsilon_\nu^2 + \epsilon_e^4 - 2\epsilon_\nu^4 - 5\epsilon_e^2 \epsilon_\nu^2) - 4\epsilon_e^2 [\epsilon_e^2 + (1 - \epsilon_\nu^2)^2] L_e \\ &\quad + 4\epsilon_\nu^4 (1 - \epsilon_e^2) L_\nu, \\ H_5 &= 0, \\ H_6 &= \frac{1}{3} R_o (1 - 5\epsilon_e^2 + 10\epsilon_\nu^2 - 2\epsilon_e^4 + \epsilon_\nu^4 - 5\epsilon_e^2 \epsilon_\nu^2) + 4\epsilon_e^4 (1 - \epsilon_\nu^2) L_e \\ &\quad - 4\epsilon_\nu^2 [\epsilon_\nu^2 + (1 - \epsilon_e^2)^2] L_\nu, \\ H_7 &= -\frac{1}{2} R_o (1 + 5\epsilon_e^2 + \epsilon_\nu^2) + 2\epsilon_e^2 \left[2(1 - \epsilon_\nu^2) + \frac{\epsilon_e^2(1 + \epsilon_\nu^2)}{1 - \epsilon_\nu^2} \right] L_e \\ &\quad + 2\epsilon_\nu^2 \left[1 - 2\epsilon_e^2 + \frac{\epsilon_e^4}{1 - \epsilon_\nu^2} \right] L_\nu, \\ H_8 &= \frac{1}{3} R_o (2 + 5\epsilon_e^2 + 5\epsilon_\nu^2 - \epsilon_e^4 - \epsilon_\nu^4 - 10\epsilon_e^2 \epsilon_\nu^2) - 4\epsilon_e^2 [\epsilon_e^2 \epsilon_\nu^2 + (1 - \epsilon_\nu^2)^2] L_e \\ &\quad - 4\epsilon_\nu^2 [\epsilon_e^2 \epsilon_\nu^2 + (1 - \epsilon_e^2)^2] L_\nu, \end{aligned}$$

where

$$L_e = \ln \frac{1 + \epsilon_e^2 - \epsilon_\nu^2 + R_o}{2\epsilon_e}, \quad L_\nu = \ln \frac{1 - \epsilon_e^2 + \epsilon_\nu^2 + R_o}{2\epsilon_\nu},$$

and $R_o = \sqrt{(1 - \epsilon_e^2)^2 - 2\epsilon_\nu^2(1 + \epsilon_e^2) + \epsilon_\nu^4}$.

As one should have expected, the partial decay width (6) does not depend on the Michel parameter ρ as well as on τ , ε , and κ , because $H_2 = H_3 = H_5 = 0$.

3 Conclusions

To most clearly represent the effect of the new tensor constant, we neglect the masses of the neutrinos and the positron, and explicitly extract the dependence of the positron energy spectrum on g_{RR}^T by setting $g_{LL}^V=1$. Then

$$d\Gamma \propto \left\{ (1 - x_e) + \frac{2}{9}\rho_o(4x_e - 3) + 2\epsilon_e \left(\frac{1 - x_e}{x_e}\eta_o + \frac{1}{x_e}g_{RR}^T \right) + \frac{(g_{RR}^T)^2}{6}(15 - 14x_e) + \frac{3\alpha}{\pi}f(x_e) \right\} x_e^2 dx_e, \quad (7)$$

where ρ_o , η_o , and g_{RR}^T can be considered as model-independent parameters, and $f(x_e)$ is a known function which describes the one-loop electromagnetic radiative correction [10]. Taking eq.(4) into account, we see that the contribution of the new terms is of the same order of magnitude as the one-loop electromagnetic correction. A more precise measurement of the energy spectrum would allow one to detect this contribution.

If we assume that all the constants but g_{LL}^V and g_{LL}^T are equal to zero, and the presented above theoretical curve (7) adequately describes the experiment, then the fit of the parameter ρ , neglecting the tensor contributions, should lead to a systematic deviation $\delta\rho = \rho - 0.75$:

$$\delta\rho = \frac{9}{64} \frac{\sum_{i=0}^n x_i^4(x_i - 0.75) \left[\epsilon_e \frac{g_{RR}^T}{x_i} + \frac{(g_{RR}^T)^2}{6}(15 - 14x_i) \right]}{\sum_{i=0}^n x_i^4(x_i - 0.75)^2}. \quad (8)$$

It is evident that the fit over the low-energy positrons $x_e < 0.75$ leads to $\delta\rho_{low} < 0$ while for the high-energy part of the spectrum $x_e > 0.75$ one obtains $\delta\rho_{high} > 0$. A diffident indication of the existence of an effect can be observed already in the data of ref.[11]. If the experimental errors were less, we would assert more assuredly that $\rho_{low} < \rho_{high}$.

The partial decay width derived from eq.(7) at $\eta_o = 0$

$$\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} \left[1 + 12\epsilon_e g_{RR}^T + 3(g_{RR}^T)^2 \right] \left[1 - \frac{\alpha}{2\pi} \left(\pi^2 - \frac{25}{4} \right) \right] \quad (9)$$

can be used for evaluating the Fermi constant G_F . Notice that if the value of g_{RR}^T is given by eq.(4), then the contribution of the new interaction is comparable to the one-loop electromagnetic radiative correction. This may lead to a perceptible change in the value of G_F .

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